Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 52764

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fifth Semester

Civil Engineering

MA 2211 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all Branches)

(Regulations 2008)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. If f(x) is discontinuous at a point x = a, then what does its Fourier series represent at that point.

2. Write the complex form of Fourier series for a function f(x) defined in $-l < x < l$.

3. State the Fourier integral theorem.

4. If F[f(x)] = F(s), then find F[f(x – a)].

5. Find the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from the relation z = a (x + y) + b.

6. Find the complete integral of $z = px + qy + \sqrt{pq}$.

7. If the ends of the string of length l are fixed and the midpoint of the string is displaced by a distance 'h' transversely and the string is released from rest, then write the initial conditions.

8. Write all possible solutions of two dimensional heat flow equation in steady state.

9. Find the Z-transform of the function f(n) = n.

10. Form the difference equation by eliminating arbitrary constant 'a' from $y_n = a.2^{n+1}$.

PART – B

(5×16=80 Marks)

11. a) i) Find the Fourier series for a function $f(x) = x + x^2$ in $(-\pi, \pi)$ and hence deduce the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ **(8)**

ii) Find the Fourier series of $y = f(x)$ up to second harmonic which is defined by the following data in $(0, 2\pi)$. **(8)**

x	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x)	1	1.4	1.9	1.7	1.5	1.2	1

(OR)

b) i) Find the half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. Hence deduce the value of $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots$ **(8)**

ii) Find the Fourier series for a function $f(x) = \begin{cases} l - x, & 0 < x \le l \\ 0, & l < x \le 2l \end{cases}$ in $(0, 2l)$. **(8)**

12. a) i) If $F_S(s)$ and $F_C(s)$ denote Fourier sine and cosine transform of a function f(x) respectively, then show that $F_C\{f(x)\sin ax\} = \frac{1}{2}\{F_S(s+a) - F_S(s-a)\}$. **(4)**

ii) Find the Fourier transform of a function $f(x) = \begin{cases} 1 - |x| \text{ for } |x| < 1 \\ 0 \text{ for } |x| > 1 \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4} dt$ by Parseval's identity. **(12)**

(OR)

b) i) State the convolution theorem for Fourier transform. **(2)**

ii) Find the Fourier sine and cosine transforms of a function $f(x) = e^{-x}$. Using Parseval's identity, evaluate :

1) $\int_0^\infty \frac{dx}{(x^2+1)^2}$ and 2) $\int_0^\infty \frac{x^2 dx}{(x^2+1)^2}$ **(14)**

13. a) i) Find the singular integral of $z = px + qy + p^2 - q^2$. **(8)**

ii) Find the general integral of $x(y^2 + z)\,p + y(x^2 + z)\,q = z\,(x^2 - y^2)$. **(8)**

(OR)

b) Solve the following equations :

i) $(D^2 - DD' - 20\,D'^2)\,z = \sin(4x - y)$. **(8)**

ii) $(D^2 - D'^2 - 3D + 3D')\,z = xy$. **(8)**

14. a) A tightly stretched string of length l has its end fastened at $x = 0$, $x = l$. At $t = 0$, the string is in the form $f(x) = kx(l - x)$ and then released. Find the displacement at any point of the string at a distance x from one end and at any time $t > 0$. **(16)**

(OR)

b) A rod of length l cm has its ends A and B kept at $0°C$ and $100°C$ respectively, until steady state conditions prevail. If the temperature at B is suddenly reduced to $0°C$ and maintained at $0°C$, find the temperature distribution $u(x, t)$ at a distance x from A at any time t. **(16)**

15. a) i) If $Z[f(n)] = F(z)$, then show that

1) $f(0) = \lim_{z \to \infty} F(z)$ and **(8)**

2) $\lim_{n \to \infty} f(n) = \lim_{z \to 1} [(z-1)\,F(z)]$

ii) Find $Z^{-1}\left[\dfrac{8z^2}{(2z-1)(4z+1)}\right]$ by using convolution theorem. **(8)**

(OR)

b) i) Find the inverse Z-transform of $\dfrac{z}{(z-1)^2(z+1)}$ by method of partial fraction. **(6)**

ii) Solve the difference equation $y(n + 2) + 4y(n + 1) + 3y(n) = 3^n$, given that $y(0) = 0$ and $y(1) = 1$, by using Z-transform. **(10)**